Convergence analysis of Lie and Strang splitting applied to operator-valued differential Riccati equations

Teodor Åberg (Lund University, Centre for Mathematical Sciences), Tony Stillfjord

Differential Riccati equations (DREs) are matrix-valued or operator-valued equations with quadratic nonlinearities that arise in many areas. Most importantly, in control theory, where their solutions provide the optimal feedback control laws for linear quadratic regulators on finite time intervals. There are numerous numerical methods for DREs, but most of these lack proper (temporal) convergence analyses. The few existing analyses consider the matrix-valued case, and rely on properties that may not necessarily hold in the operator case. This is problematic when the matrix-valued DRE arises from a spatially discretized operator-valued DRE, corresponding to the control of a partial differential equation. In this case, refining the spatial discretization increases the problem dimension, and might cause the temporal errors to grow uncontrollably.
In view of this, we provide rigorous convergence analyses of two numerical time-stepping methods, the Lie and Strang splitting schemes, when applied to operator-valued DREs. We show that they achieve the standard orders of convergence under different types of (low) regularity assumptions. Essentially, either the initial condition or the operator giving rise to the nonlinearity should be smoothing to a certain degree, but not necessarily both. We illustrate these theoretical results with several numerical experiments on DREs arising from the control of partial differential equations.

[link to pdf] [back to Numdiff-17]