First-kind Galerkin BEM for the Hodge-Helmholtz equation

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We are interested in exterior boundary value problems (BVP) for the Euclidean Hodge-Helmholtz operator $-\Delta_{HH} := \mathbf{curl}\,\mathbf{curl} - \eta\nabla\mathrm{div} - \kappa^2$, which is closely related to Maxwell equations in frequency domain. In order to solve these exterior BVPs, we consider the corresponding first-kind boundary integral equations, which were derived and analyzed in [2]. It is worth noticing that the boundary integral operators (BIOs) for Hodge-Helmholtz induce bounded and coercive sesquilinear forms in their natural energy trace spaces, and one can establish the unique solvability of the related first-kind boundary integral equations. However, the situation changes when the wavenumber κ is zero, i.e., for the Hodge-Laplacian. Then, the related BIOs feature kernels whose dimensions are linked to fundamental topological invariants of the domain Ω. Moreover, Galerkin discretization does not affect the dimensions of these kernels [3].
In this talk, we pursue the Galerkin discretization of the variational formulations in [2] and we provide numerical experiments for these boundary integral equations using Bempp [1]. We validate our implementation using a new Calderón residual technique. Then, we compare the eigenvalues related to the equivalent saddle point formulation for $\kappa = 0$ with those found in [3] and also present the spectra for small wave numbers κ and discuss its numerical consequences.

References

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