Splitting methods for nonlinear evolution equations

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Exponential operator splitting methods constitute a favourable class of time integration methods for various kinds of linear and nonlinear evolution equations. They rely on the presumption that the defining right-hand side comprises two (or more) operators

$$u(t) = F\Big(u(t)\Big) = F_1\Big(u(t)\Big) + F_2\Big(u(t)\Big) \,, \quad t \in (0, T) \,,$$

and that the numerical approximation of the associated subproblems

$$u(t) = F_1\Big(u(t)\Big) \,, \quad u'(t) = F_2\Big(u(t)\Big) \,, \quad t \in (0, T) \,,$$

is significantly simpler compared to the numerical approximation of the original problem. Under these premises, their excellent behaviour with respect to stability, accuracy, and the preservation of conserved quantities has been confirmed by a remarkable amount of contributions.
In my talk, I will review well-known achievements and recent advances on exponential operator splitting methods. As fundamental test problems, I will study Gross–Pitaevskii equations modelling Bose–Einstein condensates, their parabolic counterparts arising in ground and excited state computations, complex Ginzburg–Landau equations having a similar structure, and high-order semilinear parabolic equations describing quasicrystalline patterns. I will contrast standard splitting schemes involving real coefficients with two alternative approaches that are based on the incorporation of complex coefficients or double commutators, respectively. Besides, I will sketch the formal calculus of Lie derivatives, which provides powerful tools regarding the design and analysis of splitting methods in the context of nonlinear evolution equations.

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