Starting Approximations for SIRK Methods Applied to Index 2 DAEs

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Implicit Runge-Kutta (IRK) methods are often used to numerically approximate solutions of stiff differential equations and differential-algebraic equations (DAEs). While these methods have very useful properties that their explicit counterparts lack, they also come at the cost of having to solve a nonlinear system of equations at every time step. In this presentation, we look at applications of IRK methods to index 2 DAEs ($y = f(t, y, z)$, $0 = g(t, y)$), henceforth referred to as specialized implicit Runge-Kutta (SIRK) methods. We are particularly interested in developing high order starting approximations for the internal stages to minimize the number of fixed-point/Newton type iterations needed to solve the nonlinear system within the desired error bounds.

Starting approximations for some of these SIRK methods have been previously studied but our new formulation greatly simplifies the analysis and further-generalizes the methods for which we can apply the approximations. We expand our starting approximations and exact internal stages (in step size h) about the current time-step rather than the previous time-step by using reverse SIRK Methods. Many of these so-called reverse methods had not been considered for index 2 DAEs before this work. Reverse SIRK methods also have many convenient relationships to the original SIRK methods they stem from, such as satisfying the same simplifying assumptions.

Our starting approximations are split into two parts: The y components and the z components. Both use linear combinations of the previous step's internal stages, while the starting approximations for Y_i's also include the previous and current time-steps. For methods satisfying the simplifying assumption $C(q)$, we are able to develop starting approximations of order q for Y_i's and order $min(q, s-1)$ for Z_i's. The coefficients for these methods are found by solving a linear system dependent on the underlying IRK coefficients.

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