Equivalent systems for differential equations with polynomially distributed delays

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We consider a delay differential equation (DDE) of the form

$$y'(t) = f\Big(y(t), \int_I y(t-\tau)\, g(\tau)\, \mathrm{d}\tau\Big),$$

which includes a distributed delay in a bounded interval $I = [\tau_{\min}, \tau_{\max}] \subset [0, \infty)$ or an unbounded interval $I = [0, \infty)$. The mapping $g : I \to \mathbb{R}_0^+$ represents a probability density function associated to some probability distribution. In the case of $I = [0, \infty)$ and using a gamma distribution, it is well known that an equivalent system of ordinary differential equations (ODEs) can be arranged. We investigate the case of a bounded interval I and a polynomial g. Important instances are a uniform distribution (g constant) and a beta distribution. We derive an equivalent system of ODEs and examine its properties. For example, the stability of stationary solutions can be analysed. For comparison, the integral in the DDE is discretised by a Gaussian quadrature, which yields a DDE with multiple constant delays. We present results of numerical computations, where initial value problems are solved.

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