New efficient linearly implicit numerical methods for stiff huge differential problems

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Partial Differential Equations (PDEs) are used for modeling various phenomena. We are interested in the efficient numerical solution of PDEs from applications, such as: a DIB (Dual-Ion Batteries) model for the formation of spatio-temporal patterns in electrodeposition in batteries [3]; models for the evolution of vegetation in environments characterized by specific conditions such as soil aridity, rainfall periodicity, and so on [5]. The spatial discretization of these models leads to huge stiff initial value problems. In this talk, we derive a new class of linearly implicit numerical methods capable of solving the mentioned problems accurately, employing reasonable computing times, and reproducing the expected Turing patterns.

The proposed methods constitute a generalization of the TASE-RK (Time-Accurate and highly-Stable Explicit Runge-Kutta) numerical schemes introduced by Bassenne et al. and Calvo et al. in 2021 [2, 4]. The latter make use of appropriate preconditioners, called TASE operators, to improve the stability of explicit RK methods. The new methods, called GTRK (Generalized TASE-RK) [1], are derived using different TASE operators for each stage of the underlying explicit RK scheme. By exploiting connections between GTRK methods and W-methods for the study of consistency, we show that it is possible to drastically reduce the number of linear systems required by classical TASE-RK schemes to reach order $p \leq 4$. Furthermore, we construct A-stable GTRK methods of order $p = 2, 3, 4$. The numerical experiments show the better efficiency of the proposed methods over the classical TASE-RK schemes, and over other linearly implicit numerical methods from the scientific literature.

References

1 L. Aceto, D. Conte, and G. Pagano. Generalized TASE Runge-Kutta methods for integrating stiff differential problems. Submitted.

2 M. Bassenne, L. Fu, and A. Mani. *Time-accurate and highly-stable explicit operators for stiff differential equations.* J. Comput. Phys., 424:Paper No. 109847, 24, 2021.

3 B. Bozzini, D. Lacitignola, and I. Sgura. *Spatio-temporal organization in alloy electrodeposition: a morphochemical mathematical model and its experimental validation.* J. Solid State Electr., 17, 467–479, 2013.

4 M. Calvo, J. I. Montijano, and L. Rández. *A note on the stability of time-accurate and highly-stable explicit operators for stiff differential equations.* J. Comput. Phys., 436:Paper No. 110316, 13, 2021.

5 D. Conte, G. Pagano, and B. Paternoster. *Nonstandard finite differences numerical methods for a vegetation reaction-diffusion model.* J. Comput. Appl. Math., 419:Paper No. 114790, 17, 2023.

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