A posteriori error bounds for pseudo parabolic problems using C_0 semigroups

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We consider the second-order pseudo parabolic equation of finding $u : [0,T] \mapsto H_0^1(\Omega)$, $\Omega \subset \mathbb{R}^d$, such that

$$\mathcal{L}\partial_t u(t) + \mathcal{M}u(t) = F(t) \quad \text{in } (0,T] \tag{1}$$

with two second order, elliptic operators $\mathcal{L}, \mathcal{M} : H_0^1(\Omega) \mapsto H^{-1}(\Omega)$ and a source function $F : [0,T] \mapsto H^{-1}(\Omega)$. Furthermore an initial condition

$$u(0) = u_0, \quad u_0 \in H_0^1(\Omega), \tag{2}$$

is given. We will assume, that the operator $\mathcal{M}$ is bounded and that $\mathcal{L}$ is both bounded and coercive.
A computable a posteriori error bound in a weighted H^1-norm for a full discretisation using the backward differential formula of order two (BDF-2 method) in time and $\mathbb{P}_2$-elements in space is derived. To do so, we leverage the C_0 semigroup, generated by the operator $\mathcal{L}^{-1}\mathcal{M}$. Furthermore we adapt elliptic reconstructions introduced by C. Makridakis and R. N. Nochetto to pseudo parabolic problems.

Given some numerical results we analyze the estimate's order, efficiency and components. Last but not least, we show that we can apply our a posteriori error bound to other time discretisations like the backward Euler and Crank Nicolson method.

References

1. Ch. Makridakis and R. H. Nochetto. Elliptic reconstruction and a posteriori error estimates for parabolic problems. *SIAM J. Numer. Anal.*, 41(4):1585–1594, 2003.

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