Structure preserving neural network-based learning Euler*'s elastica and constitutive model discovery for myocardium*

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In the context of structure preserving neural network-based learning methods, two approaches are presented in the mechanical setting: approximation of Euler's elastica used for modelling of flexible endoscopes, and automated constitutive model discovery for human myocardium. We demonstrate that we can successfully train the networks, on the one hand, to preserve structural invariants, like e.g. the inextensibility constraint of the predicted trajectories [1], and, on the other hand, to satisfy thermodynamic consistency of the myocardial stress-strain behaviour [2].

Starting from a data set of solutions of the discretised static equilibria of Euler's elastica in [1], the neural networks is trained to produce trajectories – firstly as continuous functions ('continuous networks') and secondly in terms of discrete configurations ('discrete network') – for unseen boundary conditions. Different ways to ensure the inextensibility constraint are considered in the supervised learning. The continuous networks preserve the structure of the problem by construction or by incorporating the constraint into the loss function. Even though the discrete neural network does not include structural information in its design or loss, it predicts trajectories accurately satisfying the inextensibility constraint.

In [2], instead of selecting a specific constitutive model a priori and fitting its parameters to data, we utilise *constitutive neural networks* preserving physically motivated constraints. The network autonomously discovers the best model and parameters to characterise the constitutive behavior of the passive human myocardium. We demonstrate that we can successfully train the network with triaxial shear and biaxial extension tests and systematically sparsify the parameter vector with L_1-regularization. Further, we show that the discovered model outperforms popular existing myocardium models and generalises well, from homogeneous laboratory tests to heterogeneous whole heart simulations.

References

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