Regularity and numerical approximation of fractional elliptic differential equations on compact metric graphs

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The fractional differential equation $L^{\beta}u = f$ posed on a compact metric graph is considered, where $\beta > 0$ and $L = \kappa^2 - \nabla(a\nabla)$ is a second-order elliptic operator equipped with certain vertex conditions and sufficiently smooth and positive coefficients κ, a. We demonstrate the existence of a unique solution for a general class of vertex conditions and derive the regularity of the solution in the specific case of Kirchhoff vertex conditions. These results are extended to the stochastic setting when f is replaced by Gaussian white noise. For the deterministic and stochastic settings under generalized Kirchhoff vertex conditions, we propose a numerical solution based on a finite element approximation combined with a rational approximation of the fractional power $L^{-\beta}$. For the resulting approximation, the strong error is analyzed in the deterministic case, and the strong mean squared error as well as the $L_2(\Gamma \times \Gamma)$-error of the covariance function of the solution are analyzed in the stochastic setting. Explicit rates of convergences are derived for all cases. Numerical experiments for $L = \kappa^2 - \Delta, \kappa > 0$ are performed to illustrate the results.

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