Approximation of the Navier-Stokes-Cahn-Hilliard System for Incompressible Two-phase Flows

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In this project we consider the dynamics of two incompressible and immiscible fluids in the convection-dominated regime at constant temperature. For the interfacial dynamics we employ a diffuse-interface approach. The Navier-Stokes-Cahn-Hilliard (NSCH) system is widely used to model the evolution of interfaces in two-phase flows. However, the fourth-order differential operator in the NSCH system results in high computational costs. Therefore, we focus on the NSCH system in a different way and suggest a low-order approximation. The incompressible NS equations are used with artificial compressibility such that the Euler-part of the NS equations is converted to a first-order system. Then, the CH equation is approximated by the Euler-Korteweg (EK) system with friction. It has been proved in (Lattanzio & Tzavaras, 2017) that solutions of the EK system with friction converge to solutions of the CH equation in the limit of infinite friction. Finally, the NS system and the EK system with friction are coupled in a thermodynamically consistent way. By applying a relaxation approach for third-order derivatives in the coupled system, we obtain a first-order system (neglecting the viscosity term). The aim of coupling is to obtain a first-order hyperbolic system of equations. To analyze the hyperbolicity of the new system, the eigenvalues of the Jacobian of the flux are computed. The characteristic analysis of the coupled system shows that it is strictly hyperbolic. Having a hyperbolic system of equations allows us to use tailored numerical methods. In order to solve the new system we use a high-order discontinuous Galerkin method. Finally, numerical experiments for merging droplets are performed with the first-order hyperbolic system and compared to corresponding results of the original NSCH system.

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