Finite element approximation of the Monge-Ampère equation

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The Monge–Ampère equation

$$\det D^2 u = f \text{ in } \Omega \quad \text{and} \quad u = g \text{ on } \partial\Omega$$

in a convex domain Ω with suitable data f, g admits a unique generalized solution in the cone of convex functions. The use of high-order methods or local mesh refinement is very desirable for the discretization of the above problem. On the other hand, a stable algorithmic realization of the finite element method is difficult to achieve due to the strong nonlinearity and the convexity constraint.

This talk discusses a regularization approach through uniformly elliptic Hamilton–Jacobi–Bellman equations. The regularized problem possesses a unique strong solution u_ε and is accessible to the discretization with finite elements. The contribution establishes locally uniform convergence of u_ε to the convex Alexandrov solution u to the Monge–Ampère equation as the regularization parameter ε approaches 0. A finite element method for the approximation of u_ε is proposed, and the regularized finite element scheme is shown to be locally uniformly convergent. Based on Alexandrov's estimate, some a posteriori error estimates are also shown.

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