Numerical preservation of stochastic dissipativity

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Standard numerical analysis for stochastic differential equations has a clear understanding of stability in the linear case or when the drift coefficient satisfies a one-sided Lipschitz condition and the diffusion term is globally Lipschitz. By looking at many applications, it is obvious that we need a deeper mathematical and numerical insight into stability of problems with non-global Lipschitz coefficients.
This talk is aimed to analyze nonlinear stability properties of θ-methods for stochastic differential equations under non-global Lipschitz conditions on the coefficients. In particular, the concept of exponential mean-square contractivity is introduced for the exact dynamics; additionally, stepsize restrictions in order to inherit the contractive behaviour over the discretized dynamics are also given. A selection of numerical tests confirming the theoretical expectations is also presented.
Moreover, we will briefly tackle current work concerning numerical dissipativity for stochastic partial differential equations.

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