A Lie group generalized-α method with improved accuracy

Martin Arnold (Martin Luther University Halle-Wittenberg), Johannes Gerstmayr, Stefan Holzinger

Lie group integrators solve initial value problems for (ordinary) differential equations on manifolds with Lie group structure. For one-step methods, the application of classical ODE time integration methods to a locally defined equivalent ODE in terms of local coordinates has become a quasi-standard. These local coordinates are elements of the corresponding Lie algebra. They are mapped by the exponential map or by the Cayley map to the Lie group itself. For typical fields of application, there are closed form expressions that allow to evaluate these coordinate maps and the right hand side of the locally defined equivalent ODE efficiently.

For multi-step methods and for the generalized-α method with its subsidiary variables, the situation is more complex since frequent re-parametrizations of the manifold need to be avoided. As a practical consequence, the corresponding Lie group methods suffer from extra local error terms that may, however, be eliminated by appropriate correction terms (V. Wieloch, M. Arnold: *BDF integrators for mechanical systems on Lie groups*, NUMDIFF-15, 2018). Recently, these modified Lie group integrators have been interpreted in terms of time derivatives of the local coordinates. In that way, the accuracy of simulation results was substantially improved (S. Holzinger, M. Arnold, J. Gerstmayr: *Improving the accuracy of Newmark-based time integration methods*, IMSD 2024, June 2024).

In the present paper, we analyse local and global errors of these modified generalized-α methods, discuss some implementation issues and present numerical test results that illustrate the improved accuracy.

[link to pdf] [back to Numdiff-17]