Approximation of SPDE covariance operators by finite elements: A semigroup approach

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R 3.28 Thu Z2 13:30-14:00

In this talk, we consider the problem of approximating the covariance operator of the mild solution to a linear stochastic partial differential equation (SPDE). An integral equation involving the semigroup of the mild solution is derived and a general error decomposition formula is proven. This formula is applied to approximations of the covariance operator of a stochastic advection-diffusion equation and a stochastic wave equation, both on bounded domains. The approximations are based on finite element discretizations in space and rational approximations of the exponential function in time. Special attention is given to the case that the covariance operator of the underlying Q-Wiener process, which drives the SPDE, has a homogeneous kernel. We derive convergence rates in the trace class and Hilbert–Schmidt norms and we explain how the properties of the kernel of the covariance of the Q-Wiener process affect these rates. Important examples of homogeneous kernels that fit into our framework include the class of Matérn kernels. Numerical simulations illustrate the results.
This presentation is based on joint work with Mihály Kovács and Annika Lang.