Overdetermined least-squares collocation for higher-index differential-algebraic equations

Roswitha März (Institut für Mathematik, Humboldt-Universität zu Berlin)

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This is again a joint effort with Michael Hanke (KTH Stockholm) and ties in with the results we both presented at NUMDIFF-15.
We are looking for an approximate solution $x_\pi \in X_\pi$ of the initial- or boundary-value problem

$$f((Dx)'(t), x(t), t) = 0,\ t \in [a, b], \quad g(x(a), x(b)) = 0.$$

The DAE in it can be of arbitrarily high index. The ansatz-space X_π consists of componentwise and piecewise polynomial functions x_π on the grid $\pi : a = t_0 < t_1 < \cdots < t_n = b$, with continuously connected part Dx_π. We use polynomials of degree $N > 1$ for the component Dx_π but for the nondifferentiated part degree $N - 1$. Introducing $M \geq N + 1$ so-called collocation nodes $0 \leq \tau_1 < \cdots < \tau_M \leq 1$ and in turn $t_{ji} = t_{j-1} + \tau_i h_j$, we form the overdetermined collocation system

$$f((Dx_\pi)'(t_{ji}), x_\pi(t_{ji}), t_{ji}) = 0,\ i = 1, \ldots M,\ j = 1, \ldots n, \quad g(x_\pi(a), x_\pi(b)) = 0,$$

which is then solved into a special least-squares sense for x_π. The procedure is inherently simple, the numerical tests are surprisingly good, but the underlying theory is quite demanding. Considering the fact that we are dealing here with an essentially ill-posed problem, it is important to implement it very carefully. Many questions are still open. We describe achievements, difficulties and surprises.