Convergence analysis of least-squares oversampled collocation for boundary element methods

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Collocation methods for boundary integral formulations of partial differential equations are simpler and cheaper to implement than Galerkin methods because the elements of the discretisation matrix are given by lower-dimensional integrals. However, in general, their application is a delicate matter: in contrast to Galerkin methods, there is no standardised convergence theory for collocation methods, and their success is sensitive to the choice of collocation points. Moreover, in the integral equation setting, collocation methods typically lead to slower convergence rates than Galerkin methods.
In this talk, we explore the extent to which the convergence properties of collocation methods for Fredholm integral equations can be improved by least-squares oversampling. We show that superlinear oversampling can enhance the convergence rates of the collocation method and reduce its sensitivity to the distribution of collocation points. In addition, we prove that linear oversampling can still lead to a substantial improvement in the error constant, even though the asymptotic convergence rate is not improved. Indeed, we will see an example where oversampling by a constant factor leads to an improvement of the error at a cubic rate in this constant, whilst incurring only a linear increase in cost. We support our analysis with several numerical examples for the two-dimensional Helmholtz equation.