Modelling and Simulation of the Convection-Diffusion Equation through Fractional Restricted Calculus of Variations

Emina Hadzialic (University of Paderborn), Sina Ober-Blöbaum,
Fernando Jimenez Alburquerque, Jacky Cresson
R 1.23 Thu Z3 13:30-14:00

Using variational formulations is a common way to characterise equations. Moreover, it is also convenient to apply variational methods in order to develop numerical schemes. In this work we are focusing on dissipative systems. Classical variational tools do not apply to these, which leads to difficulties in the numerical study of the corresponding equations. Thus, we are proposing to look for a generalised variational setting, a new kind of fractional calculus of variations, called ”restricted calculus of variations”, allowing to deal with these equations. More precisely, we define a phase space including the fractional derivative of curves and, furthermore, we use a particular restriction upon the admissible variations. Moreover, a variational discretisation yields particular numerical schemes, called variational integrators, which, thanks to their powerful structure-preserving properties, have already shown superior performance in different scenarios, like conservative systems. In this talk we are going to introduce our preliminar continuous results about how to obtain the well-known convection-diffusion equations employing this approach. In this way, we are able to provide a fractional Lagrangian formulation of the convection-diffusion equation. We shall show that the fractional variational formulation leads to a family of fractional convection-diffusion equations. In other words, the method provides a more general expression. We will finish the presentation by showing some numerical results of our implementation.