Recent advances in deterministic and stochastic numerics for evolutive problems

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R 3.07, R 3.28@ Fri Z1 08:10-09:00

This contribution regards the numerical solution of evolutionary problems related to natural phenomena and physical processes modelled by functional equations of various type, with specific characteristics such as: stiffness, oscillations, presence of memory together with stochastic terms, by focusing especially on stability properties of the methods. We will describe the construction of adapted numerical methods, that is, strongly oriented to the problem and with excellent stability properties, where the stability will be also interpreted as preservation of the intrinsic qualitative characteristics of the problem itself.
For the numerical solution of stiff problems, originating for example from convection and diffusion terms in Partial Differential Equations (PDEs), methods with good stability properties can be obtained by exploiting the usage of the Jacobian matrix of the problem [1, 5]. Recent works on Time-Accurate and Highly-Stable (TASE) operators [2] show as the employment of these operators, together with explicit Runge-Kutta methods, can lead to A-stable schemes with a considerable gain in efficiency. We will focus our attention on the class of explicit and parallel peer methods [8, 9] combined with TASE operators, describing the derivation of highly stable numerical methods.
For problems with oscillating solution, we will show how the use of non-polynomial bases reveals a powerful tool for adapting numerical methods to the known behavior of the solution [3, 4, 7].
For problems with memory and influenced by stochastic phenomena, modeled by stochastic Volterra integral equations, the focus will be on the construction of numerical methods that can inherit good stability properties from the methods for stochastic differential equations [6].
The presented results have been obtained in collaboration with Beatrice Paternoster, Leila Moradi and Giovanni Pagano (University of Salerno), Raffaele D'Ambrosio (University of L'Aquila), Liviu Ixaru (Institute of Physics and Nuclear Engineering, Bucharest), Fakhrodin Mohamadi (University of Hormozgan, Iran), Ali Abdi (University of Tabriz, Iran).

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