On thermodynamically compatible schemes for turbulent shallow water flows

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R 1.23 Mon Z3 14:50-15:15

In this talk we will present a new reformulation of the first order hyperbolic model for unsteady turbulent shallow water flows recently proposed by *Gavrilyuk et al. 2018.* As a novelty, a novel formulation is proposed introducing a new evolution variable that guarantees the trace of the discrete Reynolds stress tensor to be always non-negative, *Busto et al. 2021.* The mathematical model is particularly challenging because one important subset of evolution equations is nonconservative and the nonconservative products also act across genuinely nonlinear fields. Therefore, we will first consider a thermodynamically compatible viscous extension of the model that is necessary to define a proper vanishing viscosity limit of the inviscid model and that is absolutely fundamental for the subsequent construction of a thermodynamically compatible numerical scheme. Then we will introduce two different, but related, families of numerical methods for its solution. The first scheme is a provably thermodynamically compatible semi-discrete finite volume scheme that makes direct use of the Godunov form of the equations resulting on the so-called discrete Godunov formalism. The new method mimics the underlying continuous viscous system exactly at the semi-discrete level and is thus consistent with the conservation of total energy, with the entropy inequality and with the vanishing viscosity limit of the model. The second scheme is a general purpose high order path-conservative ADER discontinuous Galerkin finite element method with *a posteriori* subcell finite volume limiter that can be applied to the inviscid as well as to the viscous form of the model. Both schemes have in common that they make use of path integrals to define the jump terms at the element interfaces. The different numerical methods are applied to the inviscid system and are compared with each other and with the scheme proposed in *Gavrilyuk et al. 2018* for different Riemann problems. Moreover, we make the comparison with a fully resolved solution of the underlying viscous system in the vanishing viscosity limit. In all cases an excellent agreement between the different schemes is achieved. We will furthermore show numerical convergence rates of ADER-DG schemes up to sixth order in space and time and present challenging test problems for the model where we also compare with available experimental data.

References

Gavrilyuk, S., Ivanova, K., Favrie, N. *Multi-dimensional shear shallow water flows: Problems and solutions.* Journal of Computational Physics 366, 252-280 (2018).

Busto, S., Dumbser, M., Gavrilyuk, S., Ivanova, K. *On Thermodynamically Compatible Finite Volume Methods and Path-Conservative ADER Discontinuous Galerkin Schemes for Turbulent Shallow Water Flows.* Journal of Scientific Computing 88, 28 (2021).