Optimal transport methods for mesh generation in non-convex domains

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R 3.07, R 3.28@ Mon Z1 11:40-12:30

r-adaptive mesh generation by using moving meshes for the computational solution of PDEs can be done effectively by using optimal transport (OT) methods. This proves to be a robust and reliable way of generating meshes with provable regularity and skewness measures. Finding the best mesh on which to solve a PDE then becomes a problem of solving a fully nonlinear time-evolving system on an appropriate manifold with appropriate boundary conditions. The regularity of the mesh then follows from rigorous estimates of the solution of the fully nonlinear problem.

In this talk I will describe the ideas behind using OT methods for mesh generation. These involve solving Monge-Ampére equations (or Monge-Ampére like equations in the case of the sphere) of the general form

$$m(\nabla\phi)H(\phi) = \theta$$

where $m(\nabla\phi)$ is a 'monitor function' related to the solution of the underlying PDE which is constructed to be a measure of the solution error.

I will then show how these OT mesh generation methods are implemented on both the plane and on the sphere by solving the Monge-Ampére or Monge-Ampére like equations by using fast quasi Newton methods.

In particular I will look at the solution of Poisson's equation in a non convex domain with re-entrant corners. Such problems are known to have singular solutions leading to large solution errors. I will show how optimal transport methods can be used to quickly generate meshes for solving these problems. Such meshes have scale independent regularity and lead to optimal error estimates despite the singularities in the solution of the PDE.