Implicit-explicit (IMEX) methods for evolutionary partial differential equations

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Several models in science, physics and engineering, are described by evolutionary systems of partial differential equations (PDEs). A vast literature exists on space and time discretization of such systems, and it is sometimes difficult to identify which is the appropriate technique to use for a particular problem. A well-known approach in the numerical solution of evolutionary problems in PDEs is the method of lines (MOL), in which a system of PDEs is first discretized in space by some suitable technique (finite difference, finite volume, finite element or discontinuous Galerkin), thus obtaining a large system of ordinary differential equations (ODEs), and then an ODE solver is used to find the desired numerical approximation to the problem. Traditionally, most systems of ODE's are classified as stiff or non-stiff, according to the ratio between the fastest and slowest time scales involved, and, accordingly, stiff or non-stiff ODE solvers are used for their numerical solution. When dealing with PDEs, there are many stiff problems, however, in which a fully implicit treatment becomes prohibitively expensive, and several terms of the system can be treated explicitly. In such cases Implicit-Explicit (IMEX) methods represent a natural approach, maintaining the desired stability properties without the computational cost of a fully implicit method. This talk will focus on IMEX methods, with particular emphasis on their application to systems of PDEs. Indeed, IMEX have shown to be a very effective tool for the numerical solution of a wide class of evolutionary problems, in several contexts: kinetic theory of rarefied gases, linear and nonlinear waves, viscoelasticity, multiphase flows, radiation hydrodynamics, traffic flows, shallow water, just to mention some examples. Furthermore, these methods are optimal ODE solvers for some classes of ODEs such as, for example, singular perturbation problems, which are commonly found in many areas of applied mathematics, including fluid dynamics and boundary value problems containing a small parameter, stiff nonlinear oscillators, or chemical kinetics with slow and fast reactions. IMEX methods of Runge Kutta and linear multistep type have been extensively employed for evolutionary PDEs in combination with a wide range of spatial discretization. They, also provide an ideal framework for the construction of asymptotic preserving methods, currently very popular in the context of fluid-dynamical limits of kinetic equations. Nowadays, these methods have become mainstream for the solution of evolutionary PDEs, in particular in the field of hyperbolic and kinetic equations. The construction, analysis and application of IMEX methods, developed in the last two decades, will be summarized and illustrated with the help of several examples, and test cases.