Numerical Dynamics of a Cable Subjected to Frictional Impact

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The dynamics of an elastic cable subjected to frictional contact are investigated numerically. The talk will address the formulation of a Finite Element formulation for the finite displacement of one cable subjected to the presence of one obstacle.

The cable is a curvilinear domain which torsion and bending are negligible compared to the axial force, termed as tension. Moreover, the cable is a structure which cannot overcome compression. The position and velocity of the centerline are given at any time t through the relation $S \to (q(S,t), v(S,t))$ where S is a curvilinear coordinate in the unstretched configuration. In the sequel, these dependencies are omitted for conciseness. We will refer to S and t differentiation with $\bullet'$ and $\dot{\bullet}$. The local form of system equations [1, 2] are given as follows:

$$\rho\dot{v} + 2cv = \left(EA\left(\|q'\| - 1\right)' \frac{q'}{\|q'\|}\right) + f \quad \text{such that } \|q'\| - 1 \geqslant 0 \tag{1}$$

It will be explained how a discrete nonlinear problem is obtained to predict the unconstrained dynamics of the cable in the following matrix form via finite element method [3] and how the latter is reformulated into a measure differential inclusion

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{q})\mathbf{v} + \mathbf{K}(\mathbf{q})\mathbf{q} = \mathbf{f} \rightsquigarrow \begin{cases} \mathbf{M}\mathrm{d}\mathbf{v} + \mathbf{C}(\mathbf{q})\mathbf{v}\mathrm{d}t + \mathbf{K}(\mathbf{q})\mathbf{q}\mathrm{d}t = \mathbf{f}\mathrm{d}t + \mathrm{d}\mathbf{r} \\ \mathbf{v}^+ = \dot{\mathbf{q}}^+ \\ \mathbf{q}(t=0) \in \mathcal{C}(t=0)\ ,\ \mathbf{v}(t=0^-) = \dot{\mathbf{q}}_0 \end{cases} \tag{2}$$

where $\mathcal{C}(t)$ is defined as the subspace where the dynamics are constrained to evolve, here due to the presence of one obstacle, which reads

$$\mathcal{C}(t) = \left\{\mathbf{q} \in \mathbb{R}^d\ ,\ \mathbf{g}(\mathbf{q}) \geqslant \mathbf{0} \text{ (vector inequality)}\right\} \tag{3}$$

The measure differential inclusion will be used to derive a time-stepping scheme called the NonSmooth Contact Dynamics methods [4]. The latter relies on an implicit scheme of low order which can handle jumps at the velocity level. A restitution coefficient, e, links the velocity befrore and after impact and the Coulomb law is used to model the friction (μ Coulomb coefficient). At each time step, the frictional contact problem is attacked via Lemke Method [5]. It will be explained how the local coordinates at contacting points yields to solve the Linear Complementarity Problem just below

$$\begin{cases} \mathbf{v}^{k+1} = \mathbf{v}^f + \widehat{\mathbf{M}}^{-1}\left(\mathbf{H}_N^\top \mathbf{r}^1 + \mathbf{H}_T^\top\left(\mu\mathbf{r}^1 - \mathbf{r}^2\right)\right) \quad , \quad \widehat{\mathbf{M}} = \mathbf{M} + h\mathbf{C} + h^2\Delta\mathbf{K} \\ \mathbf{u}_N^1 = \mathbf{H}_N\mathbf{v}^f \ , \ \mathbf{u}_N^0 = \mathbf{H}_N\mathbf{v}^k \ , \ \mathbf{u}_T^1 = \mathbf{H}_T\mathbf{v}^f \ , \ \mathbf{u}_T^0 = \mathbf{H}_T\mathbf{v}^k \\ \qquad \text{For all } \alpha \text{ such that } \left(\mathbf{g}(\mathbf{q}^k + h\mathbf{v}^f)\right)_\alpha \leqslant 0 : \\ \mathbf{0} \leqslant \begin{bmatrix} \widehat{\mathbf{W}}_{NN} + \mu\widehat{\mathbf{W}}_{NT} & -\widehat{\mathbf{W}}_{NT} & \mathbf{0} \\ -\widehat{\mathbf{W}}_{TN} - \mu\widehat{\mathbf{W}}_{TT} & \widehat{\mathbf{W}}_{TT} & \mathbf{I} \\ 2\mu\mathbf{I} & -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{r}_\alpha^1 \\ \mathbf{r}_\alpha^2 \\ (\mathbf{u}_T^1)^+ \end{bmatrix} + \begin{bmatrix} (\mathbf{u}_N^1)_\alpha + e\,(\mathbf{u}_N^0)_\alpha \\ -\,(\mathbf{u}_T^1)_\alpha \\ \mathbf{0} \end{bmatrix} \perp \begin{bmatrix} \mathbf{r}_\alpha^1 \\ \mathbf{r}_\alpha^2 \\ (\mathbf{u}_T^1)^+ \end{bmatrix} \geqslant \mathbf{0} \\ \qquad \text{For all } \alpha \text{ such that } \left(\mathbf{g}(\mathbf{q}^k + h\mathbf{v}^f)\right)_\alpha > 0 : \\ \mathbf{r}_\alpha^1 = \mathbf{0} \quad , \quad \mathbf{r}_\alpha^2 = \mathbf{0} \end{cases} \tag{4}$$

where $\mathbf{v}^f$ is the unconstrained velocity predicted by FEM and $(\mathbf{u}_T^1)^+$ is the positive part of $\mathbf{u}_T^1$. We used a modified Delassus operator given by the following expression:

$$\widehat{\mathbf{W}}_{NN} = \mathbf{H}_N\widehat{\mathbf{M}}\mathbf{H}_N \ , \ \widehat{\mathbf{W}}_{NT} = \mathbf{H}_N\widehat{\mathbf{M}}\mathbf{H}_T \ , \ \widehat{\mathbf{W}}_{TN} = \mathbf{H}_T\widehat{\mathbf{M}}\mathbf{H}_N \ , \ \widehat{\mathbf{W}}_{TT} = \mathbf{H}_T\widehat{\mathbf{M}}\mathbf{H}_T \tag{5}$$

Some applications will be presented for belt-pulley systems and for the vibration of a cable subjected to the presence of an obstacle ; Systems are depicted in Figure 1.

References

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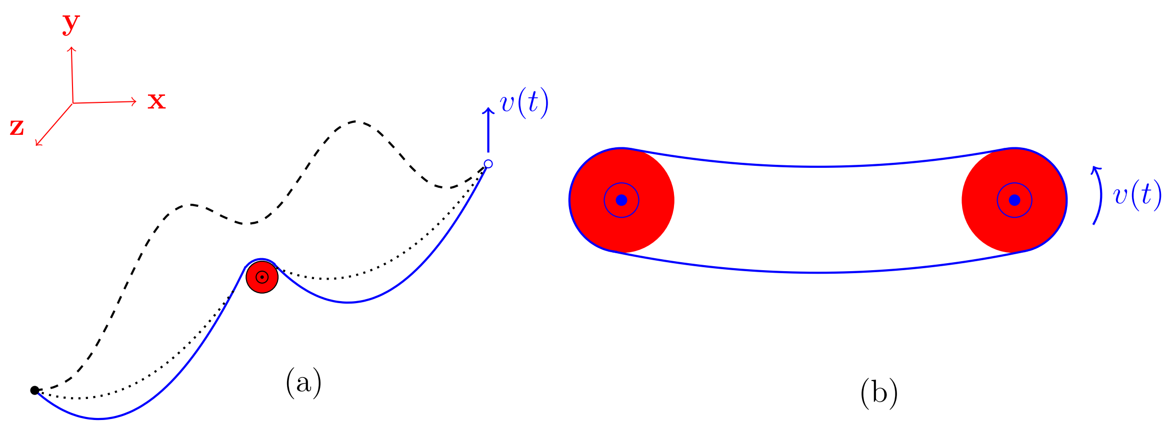

Figure 1: (a) Cable with moving end-support subjected to the presence of a circular obstacle - (b) Belt-pulley system

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