Divergence-free finite element methods for an inviscid fluid model

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R 3.07 Thu Z1 14:00-14:30

In this talk I will review some recent results [1,2] on the stabilisation of linearised incompressible inviscid flows (or, with a very small viscosity). The partial differential equation is a linearised incompressible equation similar to Euler's equation, or Oseen's equation in the vanishing viscosity limit. In the first part of the talk I will present results on the well-posedness of the partial differential equation itself. From a numerical methods' perspective, the common point of the two worksis the aim of proving the following type of estimate:

$$\|\boldsymbol{u}-\boldsymbol{u}_h\|_{L^2} \leq C\, h^{k+\frac{1}{2}} |\boldsymbol{u}|_{H^{k+1}}, \tag{1}$$

where $\boldsymbol{u}$ is the exact velocity and $\boldsymbol{u}_h$ is its finite element approximation. In the estimate above, the constant C is independent of the viscosity (if the problem has a viscosity), and, more importantly, independent of the pressure. This estimate mimics what has been achieved for stabilised methods for the convection-diffusion equation in the past. Nevertheless, up to the best of our knowledge, had only been achieved for Oseen's equation using equal-order elements, and assuming a (very) regular pressure.

I will first present results of a discretisation using $H(div)$-conforming spaces, such as Raviart-Thomas, or Brezzi-Douglas-Marini, where an estimate of the type (1) is proven (besides an optimal estimate for the pressure). In the second part of the talk I will move on to H^1-conforming divergence-free elements, with the Scott-Vogelius element as the prime example. In this case, dueto the H^1-conformity, the need of an extra control of the vorticity equation, and some appropriate jumps, appears. So, a new stabilised finite element method adding control on the vorticity equation is proposed. The method is independent of the pressure gradients, which makes it pressure-robust and leads to pressure-independent error estimates such as (1). Finally, some numerical results will be presented and the present approach will be compared to the classical residual-based SUPG stabilisation.

References :
[1] Barrenechea, G.R., Burman, E., and Guzmán, J.: *Well-posedness and H(div)-conforming finite element approximation of a linearised model for inviscid incompressible flow.* **Mathematical Models and Methods in Applied Sciences (M3AS)**, **30**(5), 847-865, (2020).
[2] Ahmed, N., Barrenechea, G.R., Burman, E., Guzmán, J., Linke, A., and Merdon, C. *A pressure-robust discretization of Oseen's equation using stabilization in the vorticity equation.* **SIAM Journal on Numerical Analysis**, to appear.