Numerical solutions of Sturm-Liouville problems with a boundary condition depending on an eigenparameter

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The following spectral problem is considered

$$-y'' + q(x)y = \lambda y,\ 0 < x < 1, \tag{1}$$

$$y(0)\cos\beta = y'(0)\sin\beta,\ 0 \le \beta < \pi, \tag{2}$$

$$y(1) = (c\lambda + d)y'(1), \tag{3}$$

where c, d are real constants and $c > 0$, λ is the spectral parameter, $q(x)$ is a real valued and continuous function over the interval $[0, 1]$.
In the current study we are mainly interested in numerical evaluation of the eigenvalues and the eigenfunctions of special eigenvalue problems such as

$$-y'' = \lambda y,\ 0 < x < 1,$$

$$y(0) = 0,$$

$$y(1) = \left(\frac{\lambda}{3} + 1\right) y'(1).$$

For this problem $\lambda_0 = \lambda_1 = 0$ is a double eigenvalue.The other eigenvalues $\lambda_2 < \lambda_3 < \dots$ are the solutions of the equation $\tan\sqrt{\lambda} = \sqrt{\lambda}\left(\frac{\lambda}{3} + 1\right)$. Eigenfunctions are $y_0 = x$, $y_n = \sin\sqrt{\lambda_n}x$ $(n \ge 2)$ and an associated function corresponding to y_0 is $y_1 = -\frac{1}{6}x^3 + Cx$, where C is an arbitrary constant. The transcendental equation $\tan\sqrt{\lambda} = \sqrt{\lambda}\left(\frac{\lambda}{3} + 1\right)$ is approximately solved to find approximate values of λ_n which then used to find formula for $y_n = \sin\sqrt{\lambda_n}x$.
We also discuss an example for which the eigenvalue is triple.