Geometric integration on Lie groups using the Cayley transformation

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This talk deals with geometric numerical integration on a Lie group using the Cayley transformation.
We investigate a coupled system of differential equations in a Lie group setting that occurs in Lattice Quantum Chromodynamics. Here, elementary particles are simulated which means expectation values of some operators are computed using the Hybrid Monte Carlo method. In this context, Hamiltonian equations of motion are solved with a time-reversible and volume-preserving integration method. Usually, the exponential function is used in the integration method as mapping from the Lie algebra to the Lie group.

The focus is put on geometric numerical integration using the Cayley transformation instead of the exponential function. The geometric properties of the method are shown for the example of the Störmer-Verlet method, both theoretically and numerically. Moreover, its advantages and disadvantages are discussed.