Model order reduction for space-adaptive simulations of unsteady incompressible flows

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We consider model order reduction for unsteady incompressible Navier-Stokes problems. A reduction of computational complexity is achieved by a Galerkin projection of the solution of a high-dimensional reference problem onto a low-dimensional subspace. We focus on subspaces generated by a proper orthogonal decomposition (POD) of space-adapted finite element snapshots. In previous works, we have investigated adaptive POD-Galerkin modeling for elliptic and parabolic problems [1, 2]. Incompressible flows pose additional challenges regarding the stability of the resulting reduced-order models and regarding the implementation of inhomogeneous initial and boundary condition.
We propose two approaches to computing reduced spaces which result in stable POD-Galerkin models. The first approach employs a projection of the adapted velocity snapshots onto a space of functions which are weakly divergence-free with respect to a pressure reference space. The resulting reduced-order model is a system of ordinary differential equations for the velocity POD coefficients. The second approach is based on separate PODs of the adapted velocity and pressure snapshots. Here, the velocity POD basis is enriched by supremizer functions computed on a reference velocity space. The stability of the velocity-pressure pair of reduced spaces is linked to the inf-sup constant of the reference discretization.
We analyze the complexity of the proposed reduced-order models, present numerical results for a benchmark problem, and compare our methods in terms of accuracy per computational cost.

References

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[2] S. Ullmann, M. Rotkvic and J. Lang, POD-Galerkin reduced-order modeling with adaptive finite element snapshots, *J. Comput. Phys.* (2016), 325:244–258, doi: 10.1016/j.jcp.2016.08.018.