

Collective integration of Hamilton PDEs

Benjamin Tapley (NTNU), Christian Offen, Robert McLachlan, Elena Celledoni, Brynjulf Owren

Many PDEs (e.g., Burgers', KdV and Camassa-Holm) can be written in the Hamiltonian formulation on a Poisson manifold; however, no general-purpose Poisson integrators are available for such systems. In [1] Poisson integrators are found for ODEs by first finding a symplectic realisation of the Poisson manifold then applying a symplectic integrator to the collective system. In this presentation we extend the work done in [1] by considering the action of the diffeomorphism group on the circle $\text{Diff}(S^1)$. The realisation is obtained as the momentum map of the cotangent lift of the group action of $\text{Diff}(S^1)$ on $C^\infty(S^1)$. In our examples we consider Burgers' and other Hamiltonian PDEs and show that by implementing symplectic integrators on a collective system, we obtain more long-term stable solutions and better preservation of the Casimir and Hamiltonian when compared to integrating the system on $\text{diff}^*(S^1)$.

[1] Robert I McLachlan, Klas Modin and Olivier Verdier, "Collective symplectic integrators" *Nonlinearity* 27 (2014) 6