Superconvergent IMEX Peer methods with A-stable implicit part

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The spatial discretization of certain time-dependent partial differential equations (e.g. advection-diffusion-reaction systems) yields large systems of ODEs where the right-hand side can be split into a stiff and a non-stiff part. We are interested in the construction of time integrators that combine the favorable stability properties of implicit methods and the low computational cost of explicit schemes. In order to guarantee consistency and thus convergence, the implicit and explicit integrator must fit together. A natural way to construct these so called implicit-explicit (IMEX) methods is to start with an appropriate implicit scheme and extrapolate it in a suitable manner. Promising candidates are implicit Peer methods as shown by Lang and Hundsdorfer [1].

In this talk, we discuss the construction of superconvergent methods with A-stable implicit part [2]. To this end, we begin with the derivation of conditions for constant step size sequences and, later, extend this notion to the setting of variable step sizes.

We start by recalling basic properties of s-stage IMEX Peer methods, such as consistency and stability, which are analyzed in detail in [1]. The main part of the talk is devoted to the concept of superconvergence, i.e. convergence of order $s + 1$, and its application to IMEX Peer methods. After a short introduction to the subject, we derive necessary and sufficient conditions on the coefficient matrices that guarantee superconvergence of the full scheme for constant step sizes. Further, we present a construction procedure for superconvergent implicit Peer methods and the subsequent extrapolation. In the second part, we discuss how the previously derived consistency conditions have to be modified such that the resulting method is superconvergent for variable step size sequences as well. In addition, we comment on the stability of these new schemes. Finally, we illustrate the advantage of the new superconvergent schemes in numerical examples and compare them with established methods, including those recently developed by Soleimani, Knoth and Weiner in [3, 4].

References

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