MRI-GARK: A Class of Multirate Infinitesimal GARK Methods

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Differential equations arising in many practical applications are characterized by multiple time scales. Multirate time integration seeks to solve efficiently multiscale systems by discretizing each component with a different, appropriate time step, while ensuring the overall accuracy and stability of the numerical solution. Multirate methods of linear multistep and Runge-Kutta type have been proposed in the literature. In a seminal paper Wensch, Knoth, and Galant (BIT Numerical Mathematics 49, 2009) developed multirate infinitesimal step (MIS) methods that discretize the slow component with an explicit Runge-Kutta method, and advance the fast component via a modified fast ODE system. Günther and Sandu (Numerische Mathematik 133, 2016) explained MIS schemes as a particular case of multirate general-structure Runge-Kutta (GARK) methods.
This work constructs a family of multirate infinitesimal GARK schemes (MRI-GARK) that extends the hybrid dynamics idea of the MIS approach. The order conditions theory and stability analyses are developed. Particular methods of order up to four are derived. Numerical results are presented confirm the theoretical findings. We expect the new MRI-GARK family to be useful for differential equations with widely disparate time scales, where the influence of the fast component on the slow one is weak.