Model order reduction for linear dynamical systems with quadratic outputs

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We consider initial value problems for linear time-invariant systems consisting of ordinary differential equations

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \qquad x(t_0) = x_0 \\ y(t) &= x(t)^\top Mx(t) \end{aligned}$$

with state variables x and inputs u. The quadratic output y represents a quantity of interest defined by a symmetric matrix M of rank k. We investigate model order reduction (MOR) for systems of high dimension. The system can be transformed into a linear dynamical system with k linear outputs, see [1]. However, many MOR methods for linear dynamical systems become inefficient or even infeasible in the case of large numbers k. Alternatively, we transform the system into a quadratic-bilinear (QB) form with a single linear output. The properties of this QB system are analyzed. We apply the MOR technique of balanced truncation from [2] to the QB system, where a stabilization is required. The solution of quadratic Lyapunov equations is traced back to the solution of linear Lyapunov equations. We present numerical results for a relevant example including a high rank k, where the two MOR approaches are compared.

References

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[2] P. Benner, P. Goyal: Balanced truncation model order reduction for quadratic-bilinear control systems. `arXiv`:1705.00160v1, April 29, 2017.