***Adapted discretization of evolutionary problems by non-polynomially fitted numerical methods***

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The talk is devoted to the discretization of selected evolutionary problems generating periodic wavefronts [5] and aims to explain the benefits gained by adapting the numerical scheme to the problem. Such an adaptation is carried out by merging the a-priori known qualitative information on the problem, as well as the structure of the vector field itself, into the numerical scheme. Particular emphasis will be given to advection-reaction-diffusion problems, for which the adaptation in space is developed by means of a finite difference scheme based on trigonometrical basis functions [3], rather than on algebraic polynomials which could strongly reduce the stepsize in order to accurately reproduce the prescribed oscillations of the exact solution. The adaptation in time takes into account that the spatially discretized problem is characterized by a vector field consisting in stiff and nonstiff terms, hence it makes sense to adopt an implicit-explicit (IMEX) time integration, which implicitly integrate only the stiff constituents, while the non-stiff part is computed explicitly. Clearly, the employ of non-polynomial basis functions makes the coefficients of the numerical method dependent on unknown parameters (i.e. the frequency of the oscillations), which need to be properly estimated [4]; the proposed estimation relies on a minimization procedure of the local truncation error that is carried out a-priori, without affecting the computational cost of the integration. A rigorous analysis on the stability and accuracy properties of the overall method is presented, together with some numerical tests, in order to highlight the effectiveness of the approach. The introduced technique also covers the case of periodic dynamics generated by evolutionary problems with memory [1, 2], discretized in terms of non-polynomially fitted quadrature methods able to accurately reproduce the oscillatory behavior with a reduced computational cost with respect to their analogous polynomial version, when a good estimate of the unknown frequency is provided. Stability issues for such a discretization are also addressed.

# References

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