Goal oriented time adaptivity using local error estimates

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When solving ODEs or PDEs, one is not always interested in the solution, but rather a quantity of interest (QoI) derived from it. Starting from an IVP (semidiscretized PDE) with solution $\mathbf{u}(t)$, we consider QoIs of the form

$$J(\mathbf{u}) = \int_0^T j(t, \mathbf{u}(t))\, dt$$

with $j : [0, T] \times \mathbb{R}^n \to \mathbb{R}$. Examples for this are the average energy production of a turbine or the drag coefficient for a vehicle.
The standard approach for controlling the error in the QoI is the dual-weighted residual method [1]. To obtain an estimate the error in the QoI, this method requires solving the given ODE (PDE) forward in time and its adjoint problem backwards in time, multiple times each, to reach a desired precision.
An alternative approach is to use time-adaptive schemes based on local error estimates [2], which require only one forward solve, but give no estimate of $J(\mathbf{u})$.
We propose a new, goal oriented and adaptive method [3] based on local error estimates. Taking the local error approach, we determine timesteps using only the quantities that are relevant for $J(\mathbf{u})$. Our error estimate consists of local error estimates in $j(t, \mathbf{u})$ and estimates of a quadrature approximation $J_h \approx J$. This gives us a new easy to implement timestep controller.
In the talk, we will present results on convergence, order of convergence and necessary requirements for these. We outline a performance analysis of the new method. In numerical test, we verify our results, show strengths and weaknesses of the method and compare it to the dual-weighted residual method and classical time-adaptivity based on local error estimates.

References

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