Doubly quasi-consistent fixed-stepsize implicit two-step peer methods for stiff ordinary differential equations

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Recently, Kulikov [1] presented the idea of double quasi-consistency, which facilitates the global error estimation and control, considerably. More precisely, a local error control implemented in such methods plays a part of global error control at the same time. Unfortunately, the property of double quasi-consistency is unavailable in the classical numerical integration formulas of Runge-Kutta or multistep type, including Nordsieck methods as well. That is why Kulikov and Weiner [2, 3, 4] extended their search for doubly quasi-consistent numerical integration tools to general linear methods and constructed the first formulas of such sort within explicit parallel peer schemes.
The focus of the present research is on accurate numerical integration formulas for treating stiff ODEs, which often arise in practice and for which explicit methods are shown to be ineffective. In this talk, we make the first step towards an accurate and efficient numerical solution of stiff ODEs and prove existence of *implicit* stepping formulas. We fulfill our investigation of double quasi-consistency within the family of fixed-stepsize implicit two-step peer schemes and construct two methods of convergence orders 3 and 4, which possess excellent stability properties. Then, these methods are equipped with an efficient local (and, hence, global) error estimation mechanism based on the embedded method approach, whose quality is assessed in numerical experiments with both nonstiff and stiff test problems with known solutions.

References

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