Hidden Dynamics

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In the last few years we have discovered a number of "illusions of noise" induced by the presence of discontinuities (e.g. switches, decisions, jumps in physical constants) in dynamical systems. When a system switches abruptly between two or more modes of behaviour, it can begin evolving along the discontinuity threshold between modes — so-called sliding dynamics. Such behaviour is usually highly robust, but it turns out that the tight constraint of the variables involved in sliding motion can unleash a frustration on other variables that makes them wild and unpredictable. Their erratic variation and sensitivity to modelling assumptions creates the illusion of underlying noise when in fact none is present.

These kinds of behaviour are changing the way we understand the dynamics that occurs at the thresholds between different regimes of behaviour. Most importantly they have implications for our very notion of determinism in systems that can switch between different modes. Hidden dynamics provides a way to open up the sites of discontinuity, and explore how far we can extend determinism. There are limits to predictability of systems as they transition between regimes, and these can manifest as arbitrary pauses in motion, or spurious illusions of noise.

The most simple and striking illusion of noise is called "jitter". If two investors trading stocks in a company seem to reach a steady trading level, jitter can send the company value into unexpected erratic fluctuations. If the supply and demand of a commodity, such as oil, are regulated to a steady level, jitter can cause the commodity price to become volatile and unstable. The basic idea can be applied to mechanical, fluid flow, electronic, or other physical systems.

In this phenomenon the devil really is in the details. A sliding mode is a dynamical solution that evolves perfectly along the threshold where a discontinuity occurs in a set of differential equations. The notion of stability of a system to perturbations at a discontinuity is not a standard one in dynamical systems, so the study of real world non-ideal switching has been a long running challenge. We now understand how the tiniest nonidealities in the description of a discontinuity can manifest themselves as enormous large scale sensitivity. Depending on the application the true system might glide smoothly along the threshold (e.g. in mechanical sticking), or it might chatter along the threshold (e.g. in electronic variable

structure control or thermostatic switching). There may be factors of time delay, hysteresis, or stochasticity in the switching process. Any of these can have a huge affect on variable not constrained by the sliding mode, but a combination of Filippov's inclusions, recent piecewise smooth dynamical theory, and singular perturbations, reveal that certain geometry constraints the erratic outcomes.
A single switch or discontinuity is very robust to non-idealities, which is part of the reason why sliding modes have been so successfully applied in electronic and mechanical control, but also in ecology, physiology, and a growing range of life science modelling. Two or more switches or discontinuities, however, become highly unstable to such perturbations, allowing them to vastly affect the outcome. Their effects can be understood within a range of behaviours known as "hidden dynamics" associated with switching. A coincidence of switches creates a sensitivity responsible for erratic or 'jittery' dynamics, which creates the illusion of underlying noise.