Linearly Implicit Rosenbrock-Wanner-Type Methods with Non-Exact Jacobian for the Numerical Solution of Differential-Algebraic Equations

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Solving differential-algebraic equations (DAEs) effectively is an ongoing topic in applied mathematics. In this context, especially regarding the computation of large networks in different fields of practical interest leads to extensive systems that must be evaluated efficiently. Due to given stiffness properties of DAEs, time-integration of such problems by linearly implicit Runge-Kutta methods in the form of Rosenbrock-Wanner (ROW) schemes is generally convenient. Compared to fully implicit schemes, they are easy to implement and avoid having to solve non-linear equations by including Jacobian information within their formulation. However, particularly when having to deal with large coupled systems, computing the exact Jacobian is costly and, therefore, proves to be a considerable drawback.

In this talk, concepts of Rosenbrock-Wanner-Type methods will be shown that allow for non-exact Jacobian entries with respect to differential and algebraic parts given when computing semi-explicit DAEs of index-1, thus enabling to apply versatile strategies that reduce computational efforts. Order conditions for realizing these methods will be presented, introducing an approach inspired by the works of Steihaug and Wolfbrandt [3] as well as Roche [2] that allows for their general derivation using an algebraic theory based on rooted trees. In this context, strategies described in [1] will be enhanced.

[1] T. Jax, G. Steinebach, Generalized ROW-Type Methods for Solving Semi-Explicit DAEs of Index-1, J. Comput. Appl. Math. 316 (2017) 213-228

[2] M. Roche, Rosenbrock Methods for Differential Algebraic Equations, Numer. Math. 52 (1988) 45-63

[3] T. Steihaug, A. Wolfbrandt, An Attempt to Avoid Exact Jacobian and Nonlinear Equations in the Numerical Solution of Stiff Differential Equations, Math. Comput. 33 (1979) 521-534