Adaptivity in model order reduction with proper orthogonal decomposition

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A crucial challenge within snapshot-based POD model order reduction for time-dependent systems lies in the input dependency. In the 'offline phase', the POD basis is computed from snapshot data obtained by solving the high-fidelity model at several time instances. If a dynamical structure is not captured by the snapshots, this feature will be missing in the ROM solution. Thus, the quality of the POD approximation can only ever be as good as the input material. In this sense, the accuracy of the POD surrogate solution is restricted by how well the snapshots represent the underlying dynamical system. If one restricts the snapshot sampling process to uniform and static discretizations, this may lead to very fine resolutions and thus large-scale systems which are expensive to solve or even can not be realized numerically. Therefore, offline adaptation strategies are introduced which aim to filter out the key dynamics. On the one hand, snapshot location strategies detect suitable time instances at which the snapshots shall be generated. On the other hand, adaptivity with respect to space enables us to resolve important structures within the spatial domain. Motivated from an infinite-dimensional perspective, we explain how POD in Hilbert spaces can be implemented. The advantage of this approach is that it only requires the snapshots to lie in a common Hilbert space. This results in a great flexibility concerning the actual discretization technique, such that we even can consider r-adaptive snapshots or a blend of snapshots stemming from different discretization methods. Moreover, in the context of optimal control problems adaptive strategies are crucial in order to adjust the POD model according to the current optimization iterate. In this talk, recent results in this direction are discussed and illustrated by numerical experiments.