

From Low-Rank to Data-Driven Gramian-Based Model Reduction

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For input-output systems, system Gramian matrices are a long established tool to quantify the properties controllability and observability. A range of associated Gramian-based model reduction methods has been developed over the last decades utilizing those attributes, starting with the classic balanced truncation for linear systems. Typically, these methods are targeted towards systems with a certain structure, such as bilinear, quadratic-bilinear or polynomial, which is exploited to efficiently obtain reduced order models using low-rank truncated Gramians. Yet, some classes of systems have complex structures that (currently) cannot be reduced by such an ansatz, for example general nonlinear control-affine systems.

For nonlinear input-output systems, system Gramian matrices can also be defined based on controllability and observability, yet their numerical computation is usually infeasible. A compromise between computability and Gramian-based model reduction for nonlinear systems is a data-driven computation, that incorporates nonlinear behavior inside an attractor and reproduces algebraic results for linear systems. These so-called empirical Gramians extend Gramian-based model reduction methods to otherwise intangible input-output systems. Their computation is based on simulated trajectories for systematic perturbations of the steady-state configuration. More recently, the empirical cross Gramian was enhanced to low-rank computation; we will demonstrate low-rank computation of the empirical controllability and observability Gramians.