A multirate implicit Euler scheme for semi-explicit DAEs of index-1: consistency and convergence analysis

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The mathematical modelling of electrical circuits often leads to large scale systems of differential equation with components which provide a very different dynamical behaviour. These systems can be integrated efficiently by multirate time integration schemes. Such multirate schemes employ inherent step sizes according to the dynamical properties of the components of the system.
In general, electrical circuits are described by differential algebraic equations. In this talk, we will apply multirate time integration schemes to semi-explicit differential-algebraic equations of index 1. We focus on systems where the algebraic constraints only occur in the slow changing components. On the basis of the implicit Euler-scheme, we will point out the details of the multirate time integration for DAEs. We will discuss different coupling approaches between the subsystems consisting of components with similar dynamic behaviour. For the analysis of the integration scheme, we follow the indirect approach and write the algebraic variables as a function of the differential variables. We will show that the order of consistency of the classical ODE multirate scheme can be maintained under suitable assumptions. We will complete our talk by an adaption of the proof of [1] that shows that by using a constant macro step-size the multirate implicit Euler scheme also converges for semi-explicit DAEs of index-1.

[1] Deuflhard, P., Hairer, E., Zugck, J., One-Step and Extrapolation Methods for Differential-Algebraic Systems. Numer. Math., 51, 1987, pp. 501-516.