

A Localized Reduced-Order Modeling Approach for PDEs with Bifurcating Solutions

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Reduced-order modeling (ROM) commonly refers to the construction, based on a few solutions (referred to as snapshots) of an expensive discretized partial differential equation (PDE), and the subsequent application of low-dimensional discretizations of partial differential equations (PDEs) that can be used to more efficiently treat problems in control and optimization, uncertainty quantification, and other settings that require multiple approximate PDE solutions. In this work, a ROM is developed and tested for the treatment of nonlinear PDEs whose solutions bifurcate as input parameter values change. In such cases, the parameter domain can be subdivided into subregions, each of which corresponds to a different branch of solutions. Popular ROM approaches, such as proper orthogonal decomposition (POD), results in a global low-dimensional basis that does not respect nor take advantage of the often large differences in the PDE solutions corresponding to different subregions. Instead, in the new method, the k-means algorithm is used to cluster snapshots so that within cluster snapshots are similar to each other and are dissimilar to those in other clusters. This is followed by the construction of local POD bases, one for each cluster. The method also can detect which cluster a new parameter point belongs to, after which the local basis corresponding to that cluster is used to determine a ROM approximation. Numerical experiments show the effectiveness of the method both for problems for which bifurcation cause continuous and discontinuous changes in the solution of the PDE.