On the zero-stability of multistep methods on smooth nonuniform grids

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In this talk we investigate zero stability on compact intervals and smooth nonuniform grids. The grid points $\{t_n\}_{n=0}^N$ are constructed as the image of an equidistant grid under a smooth deformation map, i.e., $t_n = \Phi(\tau_n)$, where $\tau_n = n/N$ and the map Φ is monotonically increasing with $\Phi(0) = 0$ and $\Phi(1) = 1$. We show that for all strongly stable linear multistep methods, there is an N^* such that a condition of zero stability is always fulfilled for $N > N^*$, provided that $\Phi \in C^2[0, 1]$. Thus zero stability is maintained whenever adjacent step sizes satisfy $h_n/h_{n-1} = 1 + \mathrm{O}(N^{-1})$. This suggests that variable step size should always be implemented using smooth step size changes.

The talk is based on the paper

G. Söderlind, I. Fekete, I. Faragó: On the zero-stability of multistep methods on smooth nonuniform grids, BIT Numer. Math., `https://doi.org/10.1007/s10543-018-0716-y`, 2018.