Minimal residual linear multistep methods

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Consider an initial value problem for the system of ODEs $y' = f(t, y)$ and suppose that we have k starting values $y_0, \ldots, y_{k-1}$ at points $\{t_j\}$ which are not necessarily equidistant. To compute $y_k \approx y(t_{k-1} + \tau)$ take an explicit linear multistep method with unknown coefficients:

$$y_k = \sum_{j=0}^{k-1} (\tau \beta_j f_j - \alpha_j y_j). \tag{1}$$

On the other hand consider the corresponding classic p-step implicit BDF formula

$$c_{k-p} y_{k-p} + \ldots + c_k y_k = \tau f_k, \quad p \leq k. \tag{2}$$

In the talk we discuss what happens if on each step of numerical integration the coefficients $\{\alpha_j, \beta_j\}$ of (1) are chosen to minimize the norm of the residual of method (2). The main focus will lie on the most tractable case of linear problems with $f(t, y) = A(t)y + b(t)$.