Splitting methods for highly oscillatory differential equations

Benjamin Dörich (Karlsruhe Institute of Technology (KIT)), Marlis Hochbruck

In this talk we consider the time integration of highly oscillatory differential equations of the form

$$y''(t) = -\Omega^2 y(t) + g(y(t))$$

which typically arise in the space discretizaion of semi linear wave equations. In contrast to the classical analysis we do not assume high regularity of the solution but only a so called finite energy condition. For "nice" functions g one can use trigonometric integrators with filter functions to obtain second order error estimates, cf. [1, Chapter 13] and references given there. However, for g representing a first order differential operator these integrators fail. For example in the linear case numerical experiments indicate very large error constants.
We show that much better results can be achieved by constructing new filter functions and adapting the techniques from the analysis in [2]. Numerical examples confirming the theoretical results are also presented.

References

[1] Hairer, E. and Lubich, C. and Wanner, G. (2006), Geometric Numerical Integration — Structure-Preserving Algorithms for Ordinary Differential Equations, Berlin: Springer.
[2] Buchholz, L. Gauckler, V. Grimm, M. Hochbruck, and T. Jahnke. Closing the gap between trigonometric integrators and splitting methods for highly oscillatory differential equations. IMA J. Numer. Anal., 38(1):57–74, 2018.