Analysis of multilevel Monte Carlo using the Milstein discretisation

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Using a simple Monte Carlo method with a numerical discretisation with first order weak convergence, to achieve a root-mean-square error of $\mathcal{O}(\epsilon)$ would require $\mathcal{O}(\epsilon^{-2})$ independent paths, each with $\mathcal{O}(\epsilon^{-1})$ timesteps, giving a computational complexity which is $\mathcal{O}(\epsilon^{-3})$. However, Giles' multilevel Monte Carlo (MLMC) approach ([1]), which combines the results of simulations with different numbers of timesteps, reduces the cost to $\mathcal{O}(\epsilon^{-2})$ under certain circumstances.
In this presentation we analyse the efficiency of the MLMC approach for different options and scalar SDEs using the Milstein discretisation, determining or bounding the order of convergence of the variance of the multilevel estimator, and hence the computational complexity of the method.

References

[1] Giles, Michael B. 2008. Multilevel Monte Carlo path simulation. *Oper. Res.*, **56**(3), 607–617.

[2] Giles, Michael B., Debrabant, Kristian, & Rößler, Andreas. 2018. *Analysis of multilevel Monte Carlo using the Milstein discretisation*. Preprint.