Analysis of splitting schemes for the stochastic Allen-Cahn equation

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The stochastic Allen-Cahn equation, with additive space-time white noise perturbation, in dimension 1, is given by the following semilinear SPDE

$$dX(t) = AX(t)dt + \big(X(t) - X(t)^3\big)dt + dW(t).$$

Since the nonlinearity $x \mapsto x - x^3$ is not globally Lipschitz continuous, the design of suitable temporal discretization scheme is delicate. We propose to use a splitting strategy, taking into account that the flow $\big(\Phi_t(z)\big)_{t\geq 0}$ of the ODE $\dot{z} = z - z^3$ is exactly known.
We study numerical schemes defined as

$$X_{n+1} = e^{\Delta t A}\Phi_{\Delta t}(X_n) + \int_{n\Delta t}^{(n+1)\Delta t} e^{(n\Delta t - t)A} dW(t),$$

(exact sampling of the stochastic convolution), or as

$$X_{n+1} = S_{\Delta t}\Phi_{\Delta t}(X_n) + S_{\Delta t}\big(W((n+1)\Delta t) - W(n\Delta t)\big)$$

with $S_{\Delta t} = (I - \Delta t A)^{-1}$ (semi-implicit discretization of the stochastic convolution).
Moment estimates, as well as strong and weak convergence rates, will be presented.
I will also present numerical simulations supporting the theoretical results.