Gramian-based Model Reduction for Classes of Nonlinear Systems

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For linear input-output systems, system Gramian matrices are a long established tool to quantify the properties controllability and observability. A range of associated Gramian-based model reduction methods has been developed over the last decades utilizing those attributes, starting with the landmark paper by Moore (1981) introducing balanced truncation in the form it has been used since then. During the last two decades, the method has also become computationally feasible for truly large-scale systems arising from discretizing systems with dynamics defined by unsteady PDEs. This is mainly due to low-rank techniques, allowing to compute the information necessary for implementing balanced truncation at almost linear complexity w.r.t. the order of the system (in contrast to the cubic complexity of traditional implementations).
We show that these techniques can also be used to define approximate balanced truncation methods for some classes of nonlinear systems. This is based on combing low-rank techniques for Gramian computation with the concept of truncated Gramians derived from the Volterra series representation of the system response. We introduce these techniques for bilinear, quadratic-bilinear, and polynomial systems. The performance of the new methods is illustrated by several numerical examples.