hp-FEM solutions for option price Bates' model

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For valuating options, several stochastic models have been developed, where several assumptions on the market are imposed. For example, Black-Scholes' model considers constant volatility and local small changes. To overcome these simplifications, Bates' model [4] includes stochastic volatility and jumps, which corresponds to the following system of stochastic differential equations

$$\begin{cases} dS_t = (\alpha - \frac{1}{2}Y_t)dt + \sqrt{Y_t}dW_1 + dq, \\ dY_t = \xi(\eta - Y_t)dt` + \theta\sqrt{Y_t}dW_2 \end{cases} \tag{1}$$

which can be reduced to a partial integro-differential equation on $\Omega \times (0,T) = (0,S_0) \times (0,1) \times (0,T)$

$$\frac{\partial C}{\partial t} + (r - q - \kappa(1)) S \frac{\partial C}{\partial S} + \frac{1}{2} y S^2 \frac{\partial^2 C}{\partial S^2} + \left[\xi(\eta - y) - \frac{1}{2}\left(\theta^2 - \rho\theta y\right)\right] \frac{\partial C}{\partial y}$$
$$\frac{1}{2}\theta^2 y \frac{\partial^2 F}{\partial y^2} + \rho\theta y S \frac{\partial^2 C}{\partial y \partial S} + \lambda \int_{-\infty}^{\infty} C\left(S\exp(x), y, t\right) W(dx) = (r + \lambda) C.$$

with boundary conditions $C(0,y,t) = 0$, $C(S_0,y,t) = S_0 - K$ and final condition $C(S,y,T) = (S-K)^+$. The conditions on y are undefined.
Based on Achdou & Tchou [1], Hilber et al. [5], [6] and Reich et al. [9], we show the variational formulation and prove a Gårding type inequality. Also localization error is obtained. We base our numerical studies on Almendral & OOsterlee [2], Ballestra & Sgarra [3] and Miglio & Sgarra [8]. We propose that hp-FEM methods [7], as special method of singularly elliptic problems, can be used to improve unstabilities of the FEM methods detected in the simplification of the splitting. Some studies on the parameters on the effect of convective part over the diffusion part are also considered.

Keywords: Stochastic option pricing models, hp-finite element method, degenerate partial integro-differential equations

Mathematics Subject Classifications (2000):35K65, 65M15, 65M60, 65N30.

References

[1] Achdou, Y., Tchou, N. *Variational analysis for the Black-Scholes equation with stochastic volatility* ESAIM: Mat. Mod. and Num. Analysis, 36/3, 373-395 (2002).

[2] Almendral, A., OOsterlee, C.W. *Numerical valuation of options with jumps in the underlying* Appl. Num. Math., 53, 1-18 (2005).

[3] Ballestra, L.V. & Sgarra, C. *The evaluation of American options in a stochastic volatility model with jumps: an efficient finite element approach* Comput. Math. Appl. 60 (2010), no. 6, 1571–1590.

[4] D. Bates *Jumps and stochastic volatility: the exchange rate processes implicit in Deutschemark options* Rev. Fin. Studies 9 (1996) 69–107.

[5] N. Hilber, Matache, A.-M., Scwab, C. *Sparse wavelet methods for option pricing under stochastic volatility* J. Comp. Finance, 8/4, 1-42 (2005).

[6] N. Hilber, N. Reich, C. Schwab & C. Winter *Numerical methods for Lévy processes* Finance Stoch (2009) 13: 471–500

[7] J. M. Melenk, *hp-Finite Element Methods for Singular Perturbations* Lecture Notes in Mathematics 1796, Springer, 2002.

[8] E. Miglio, E. & C. Sgarra *A finite element discretization method for option pricing with the Bates model* S$\vec{e}$MA J. No. 55 (2011), 23–40.

[9] N. Reich, C. Schwab & C.Winter On Kolmogorov equations for anisotropic multivariate Lévy processes Finance Stoch (2010) 14: 527–567.