Convergence analysis of the Modified Craig-Sneyd scheme for two-dimensional convection-diffusion equations with nonsmooth initial data

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In this paper we consider the Modified Craig–Sneyd (MCS) scheme which forms a prominent time stepping method of the Alternating Direction Implicit type for multidimensional time-dependent convection-diffusion equations with mixed spatial derivative terms. When the initial function is nonsmooth, which is often the case in for example financial mathematics, application of the MCS scheme can lead to spurious erratic behaviour of the numerical approximations. We prove that this undesirable feature can be resolved by replacing the very first MCS timesteps by several (sub)steps of the implicit Euler scheme. This technique is often called Rannacher time stepping. We derive a useful convergence bound for the MCS scheme combined with Rannacher time stepping when it is applied to a model two-dimensional convection-diffusion equation with mixed-derivative term and with Dirac-delta initial data. Ample numerical experiments are provided that show the sharpness of our obtained error bound.