TVD-based Finite Volume Methods for Sound-Advection-Buoyancy Systems

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The simulation of atmospheric dynamics is an important issue in Numerical Weather Prediction. It relies on the numerical solution of the Euler equations. These equations exhibit phenomena on different temporal scales. In the lower troposphere sound waves propagate approximately ten times faster than the advective waves. After remarks on the historical development of numerical weather prediction we present multirate infinitesimal step (MIS) schemes based on a finite volumes spatial discretization with different treatment of slow and fast processes in the time discretization. An approach to overcome the CFL restriction caused by sound waves are split-explicit methods. Through multirate techniques the terms relevant for sound waves are integrated by small time steps with a cheap time integration procedure, whereas the slow processes are solved by an underlying Runge-Kutta method using a larger macro step size.
The analysis of these methods is based on the interpretation as an exponential or Lie group integrator. By assuming an exact solution of the fast waves with frozen coefficients for the slow waves order conditions for our multirate infinitesimal step methods are derived. Stability is discussed with respect to the linear acoustics equation.
We construct methods based on TVD-RK schemes by different search and optimization procedures. For the established RK3 time stepping scheme the stability bound with respect to the sound CFL number is approximately 3. For our methods this bound extends up to 12. Numerical simulation results for established benchmark problems are presented. The theoretically predicted properties are confirmed by the experiments.