Adaptive timestep control for high order implicit Runge-Kutta methods

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It is possible to construct fully implicit Runge–Kutta methods such as Gauß-Legendre, Radau-IA, Radau-IIA, Lobatto-IIIA, -IIIB, and -IIIC methods of arbitrary high order of convergence. The aim of this paper is to find a new adaptive time stepping for these classes. Adaptive time step control with embedding is well-known for Runge–Kutta methods, and therefore new embedded methods of order $s-1$ for the above classes of fully implicit Runge–Kutta methods are constructed.
Since these fully implicit methods need the solution of a huge non-linear system of equations different approaches for non-linear equations are discussed and compared. It can be observed that non-linear solvers such as the usually used simplified Newton method have a step size restriction if they are used in the solution process of higher order FIRK methods.
We apply our new methods on some ODEs, DAEs and PDEs to show that this approach can be a more efficient approach than using lower order methods (see [1])

Literature:

[1] Rang, Joachim and Niekamp, Rainer. *A component framework for the parallel solution of the incompressible Navier-Stokes equations with Radau-IIA methods.* Int. J. Numer. Meth. Fluids 78 (5), pp. 304-318, 2015.