Unique solutions of initial value problems for mechanical systems with redundant unilateral constraints

Manuela Paschkowski (Martin Luther University Halle–Wittenberg), Martin Arnold

Making use of Langrange multiplier methods the dynamics of interconnected rigid or flexible bodies under certain inequality constraints can be modeled as a system of second order differential equations coupled with a complementarity problem

$$\begin{aligned} M(q)\ddot{q} &= f(q,\dot{q},t) + G^T(q)\lambda, \\ 0 \leq \lambda &\perp g(q) \geq 0. \end{aligned}$$

Here, the mass matrix $M(q) \in \mathbb{R}^{n\times n}$ is supposed to be symmetric, positive definite. A common assumption is furthermore that the active constraints $g(q) \geq 0$ are independent which means that the Jacobian matrix $G(q) := \frac{\partial}{\partial q}\, g(q) \in \mathbb{R}^{m\times n}$ has full rank. But many application models (e.g. of vehicle construction, biomechanics or robotics) deal with dependent (redundant) constraints. One option is to identify and eliminate these terms. However, if the complexity and the computational effort of the implementation should be optimized, the use of redundant constraints cannot be avoided. Using existence results for generalized complementarity problems and measure differential inclusions we generalize the proof of existence of solutions to the case with redundant unilateral constraints. Furthermore we will discuss existence and uniqueness of solutions in the case of singular mass matrices.